

## Intermediate Algebra

### Review of Applications

1. Leigh is paid time-and-a-half for hours worked in excess of 40 hours and double-time for hours worked on Sunday. If Leigh had gross weekly wages of \$342 for working 50 hours, 4 of which were on Sunday, what is her regular hourly rate?

The word that represents the equal sign in this problem is “had”

English Equation: Gross Weekly wages were \$342, or wages = 342.

We are looking for her regular hourly rate, Let  $x$  = Leigh’s regular hourly rate. Then  $1.5x$  is the overtime rate and  $2x$  is the Sunday rate.

Now, we are told that she worked a total of 50 hours, 4 on Sunday. That leaves 6 overtime hours and 40 regular hours. So her week’s wages are:  $40x + 6(1.5x) + 4(2x)$ .

Now we are ready to write a mathematical equation.

$$40x + 6(1.5x) + 4(2x) = 342$$

$$57x = 342$$

$$x = 6$$

Leigh’s regular hourly wage is \$6.

2. Kate requires \$6000 per year in extra income to pay for Christian schooling for her children. She has \$50,000 to invest and can invest in B-rated bonds paying 15% per year or in a CD paying 7% per year. How much should be invested in each to realize exactly \$6000 in interest per year?

	Interest Rate %	Principal \$	Interest Earned \$
Investment 1	0.15	$x$	$0.15x$
Investment 2	0.07	$50000 - x$	$0.07(50000 - x)$
Total		50000	6000

$$0.15x + 0.07(50000 - x) = 6000$$

$$0.15x + 3500 - 0.07x = 6000$$

$$0.08x = 2500$$

$$x = 31250$$

Kate should invest \$31250 in B-rated bonds and \$18750 in the CD.

3. You have 5 hours to take a kayak trip on a beautiful river with a 2 mile per hour current. You park your car at the landing and launch your kayak. You paddle at a rate of 5 miles per hour in still water. For how long can you travel upstream before you have to turn around so you are back at your car in exactly 5 hours?

	Rate mph	time hours	Distance miles
Upstream	3	t	3t
Downstream	7	5 - t	7(5 - t)
Total		5	

Distances are equal so we get the equation:

$$3t = 7(5 - t)$$

$$3t = 35 - 7t$$

$$10t = 35$$

$$t = 3.5$$

You must turn around in exactly 3.5 hours.

4. John can mow the yard in 2 hours using the riding mower and Bill can mow the yard in 5 hours using the push mower. If both men work together, how long will it take to mow the yard?

Reword to hear the fractions: John mows one yard per 2 hours  
 Bill mows one yard per 5 hours  
 Together they mow 1 yard per x hours.

From our sentences, the parts add to the whole, so we get the equation:

$$\frac{1}{2} + \frac{1}{5} = \frac{1}{x}$$

$$\frac{7}{10} = \frac{1}{x}$$

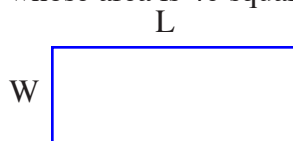
$$\frac{10}{7} = x$$

$$x \approx 1.43$$

It will take about 1 hour and 26 minutes to mow the lawn if they work together.

5. Find the dimensions of a rectangle whose area is 40 square feet and whose perimeter is 26 feet.

Label the width and length, W and L.



English Equations: Area = 40 ft<sup>2</sup> and Perimeter = 26 ft

Let's write these sentences in mathematics and work our way to one equation with only one variable.

$$L \cdot W = 40 \quad 2L + 2W = 26$$

If we solve the first equation for W, we will get a fraction with a variable in the denominator. If we solve the second equation for W, we get a fraction, but it only has a 2 in the denominator and it reduces. This is the best choice.

From equation 2:  $2W = 26 - 2L$

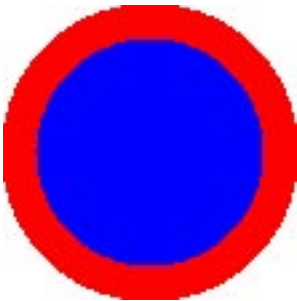
$$W = 13 - L$$

Now substitute this into the first equation for W.

Equation 1 becomes:  $L(13 - L) = 40$  This is a quadratic equation, get one side = 0.  
 $13L - L^2 = 40$   
 $0 = L^2 - 13L + 40$   
 $L^2 - 13L + 40 = 0$  Factor the non-zero side.  
 $(L - 8)(L - 5) = 0$  Set each factor = 0 and solve  
 $L = 8$  or  $L = 5$

The dimensions of the rectangle are 8 ft by 5 ft.

6. A pool in the shape of a circle measures 10 feet across. One cubic yard of concrete is to be used to create a circular border of uniform width around the pool. If the border is to have a depth of 4 inches, how wide will the border be?



We know that the radius of the pool (inner circle) is 5 feet.

Let the width of the border be  $x$  feet, then the radius of the outer circle is  $5 + x$  feet.

From geometry, we know that volume is area of the base times the height. Our base is the red circle. To find its area we will subtract the inner circle's area from the outer circle's area.

$$\begin{aligned} \text{Area of base} &= \pi(x + 5)^2 - \pi(5^2) && \text{Area of a circle} = \pi r^2 \\ &= \pi(x^2 + 10x + 25) - 25\pi \\ &= \pi x^2 + 10\pi x + 25\pi - 25\pi \\ &= \pi x^2 + 10\pi x && \text{in square feet} \end{aligned}$$

The border is to be 4 inches deep. 4 inches =  $\frac{1}{3}$  feet

$$\begin{aligned} \text{Volume of border} &= \frac{1}{3}(\pi x^2 + 10\pi x) \\ &= \frac{1}{3}\pi x^2 + \frac{10}{3}\pi x && \text{in cubic feet} \end{aligned}$$

The border will take 1 cubic yard of concrete, so the volume of the border is  $1 \text{ yd}^3$ .

$$1 \text{ yd}^3 = \frac{1 \text{ yd}^3}{1} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 27 \text{ ft}^3 \quad \text{We need cubic feet so units match.}$$

Let  $x$  = the width of the border in feet

$$\pi x^2 + 10\pi x = 27 \quad \text{This is a quadratic equation.}$$

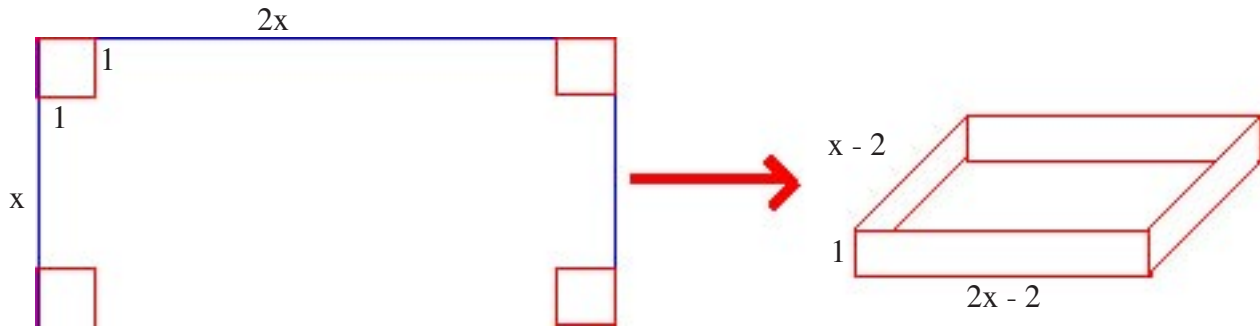
$$\pi x^2 + 10\pi x - 27 = 0 \quad \text{Use the quadratic formula.}$$

$$x = \quad \text{This is ugly, but since it is a word}$$

$$x \approx 2.13 \text{ or } x \approx -12.13 \quad \text{Negative numbers are not reasonable for widths.}$$

The border will be about 2.13 feet wide.

7. An open box is to be constructed from a rectangular sheet of metal whose length is twice its width by removing a 1 foot square from each corner and turning up the edges. If the box is to hold 4 cubic feet, what should be the dimensions of the sheet of metal?



The volume of the box in the picture is  $1(2x - 2)(x - 2)$  cubic feet

The volume of the box is supposed to equal 4 cubic feet.

Let  $x$  = the width of the sheet of metal.

$$1(2x - 2)(x - 2) = 4 \quad \text{This is a quadratic equation.}$$

$$2x^2 - 6x + 4 = 4 \quad \text{Get one side equal to zero.}$$

$$2x^2 - 6x = 0 \quad \text{Factor}$$

$$2x(x - 3) = 0 \quad \text{Set each factor = 0 and solve.}$$

$$x = 0 \text{ or } x = 3 \quad \text{Zero is not a reasonable answer.}$$

The dimensions of the sheet of metal will be 3 feet by 6 feet.

8. Tom and Jane can weed the garden in 3 hours. When working alone, Tom takes 1 hour longer to weed the garden than Jane takes when she works alone. How much time does each take to weed the garden alone?

Let  $x$  = amount of time Jane takes to weed the garden alone.

Jane weeds one garden per  $x$  hours.

Tom weeds one garden per  $(x + 1)$  hours.

Together they weed 1 garden per 3 hours.

$$\frac{1}{x} + \frac{1}{x+1} = \frac{1}{3} \quad \text{Write the equation.}$$

$$3x(x+1)\left(\frac{1}{x} + \frac{1}{x+1}\right) = 3x(x+1)\frac{1}{3} \quad \text{Multiply each side by the LDC}$$

$$3(x+1) + 3x = x(x+1) \quad \text{Simplify}$$

$$3x + 3 + 3x = x^2 + x$$

$$x^2 - 5x - 3 = 0 \quad \text{Quadratic equation, get one side = 0.}$$

$$x = \frac{5 \pm \sqrt{25 + 12}}{2} \quad \text{Quadratic formula}$$

$$x = \frac{5 \pm \sqrt{37}}{2}$$

$$x \approx 5.54 \quad \text{The negative answer is not in the domain.}$$

It takes Jane about 5 hours to weed the garden working alone, and it takes Tom about 6 hours.

9. An air rescue plane averages 300 miles per hour in still air. It carries enough fuel for 5 hours of flying time. If, upon takeoff, it encounters a wind of 30 miles per hour and the direction of the airplane is with the wind in one direction and against it in the other, how far can it fly and return safely? (Assume the wind remains constant.)

	Rate	Time	Distance	
with wind	$300 + 30$	$t$	$330t$	With wind, the plane will go faster
against wind	$300 - 30$	$5 - t$	$270(5 - t)$	Against wind, the plane will go slower.
total		5		

The only total we have is in the time column, but that equation is an identity. We need to find another equation. It will come from the distance column and the fact that the distance there must equal the distance back.

$$330t = 270(5 - t) \quad \text{Write the equation. It is linear.}$$

$$330t = 1350 - 270t \quad \text{Simplify}$$

$$600t = 1350 \quad \text{Isolate } t$$

$$t = 2.25$$

The plane can fly 742.5 miles and return safely.

10. A dietician at General Hospital wants a patient to have a meal that has 47 grams of protein, 58 grams of carbohydrates, and 630 milligrams of calcium. The hospital food service tells the dietician that the dinner for today is pork chops, corn on the cob, and 2% milk. Each serving of pork chops has 23 grams of protein, 0 grams of carbohydrates, and 10 milligrams of calcium. Each serving of corn on the cob contains 3 grams of protein, 16 grams of carbohydrates, and 10 milligrams of calcium. Each glass of 2% milk contains 9 grams of protein, 13 grams of carbohydrates and 300 milligrams of calcium. How many servings of each food should the dietician provide for the patient?

	grams protein	grams carbohydrates	milligrams calcium
pork chop	23	0	10
corn	3	16	10
milk	9	13	300
total	47	58	630

Let  $x$  = # servings of pork chops.  $23x + 3y + 9z = 47$

$y$  = # serving of corn on the cob.  $16y + 13z = 58$

$z$  = # servings of 2% milk.  $10x + 10y + 300z = 630$

Write the matrix

$$\begin{bmatrix} 23 & 3 & 9 & 47 \\ 0 & 16 & 13 & 58 \\ 10 & 10 & 300 & 630 \end{bmatrix}$$

MATRIX[A] 3 x 4

-3	9	47	1
-16	13	58	1
-10	300	630	1

3, 4=630

rref([A])

1	0	0	11
0	1	0	21
0	0	1	21

The dietician should provide one serving of pork chops, 2 servings of corn on the cob, and 2 servings of 2% milk.