Intermediate Algebra Name ________________________________

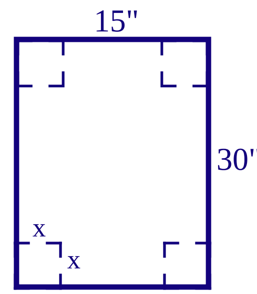
Practice Final

1. Find all values of *b* for which $x^2 + bx - 20$ can be factored.

Find all numbers that add to "b" and multiply to -20.

$-20 = -1 \cdot 20$	$-20 = 1 \cdot -20$	$-20 = -2 \cdot 10$	$-20 = 2 \cdot -10$	$-20 = -4 \cdot 5$	$-20 = 4 \cdot -5$
$b = 19$	$b = -19$	$b = 8$	$b = -8$	$b = 1$	$b = -1$

2. An open box is made by cutting squares out of the corners of a piece of cardboard that is 15 inches by 30 inches. If the edge of each cut-out square is *x* inches,



a) Write an expression involving *x*, for the volume of the box.

Volume: $V(x) = x(15 - 2x)(30 - 2x)$

b) What are the dimensions of the box with the maximum volume?

Get a good graph of the function and then find the maximum.

A good window is (0, 8, 1, 0, 1000, 100)

Maximum at $x = 3.17$ $y = 649.52$

The dimensions of the box should be about 3.17" by 8.66" by 23.66" for a volume of 649.52 in^3.

3. $f(x) = 2x^2 + 3x$

Evaluate this function for the following input.

a) $f(t) = 2t^2 + 3t$

b) $f(t + 1) = 2(t + 1)^2 + 3(t + 1)$

$2(t^2 + 2t + 1) + 3t + 3$

$2t^2 + 4t + 2 + 3t + 3$

$2t^2 + 7t + 5$

c) $f(t) + 1 = 2t^2 + 3t + 1$

d) $f(t + 1) - f(t) = (2t^2 + 7t + 5) - (2t^2 + 3t)$

$= 4t + 5$

e) $f(-x) = 2(-x)^2 + 3(-x)$

$2x^2 - 3x$

f) $-f(x) = -(2x^2 + 3x)$

$-2x^2 - 3x$

g) $f(x + h) = 2(x + h)^2 + 3(x + h)$

$2(x^2 + 2xh + h^2) + 3x + 3h$

$2x^2 + 4xh + 2h^2 + 3x + 3h$

h) $f(x + h) - f(x) = (2x^2 + 4xh + 2h^2 + 3x + 3h) - (2x^2 + 3x)$

$4xh + 2h^2 + 3h$

4. Find the equation of the line that passes through the points (7, 2) and (-3, 14).

Slope of line segment = $\frac{14-2}{-3-7} = \frac{12}{-10} = -\frac{6}{5}$

Equation of line: $y - 2 = -\frac{6}{5}(x - 7)$

$y - 2 = -\frac{6}{5}x + \frac{42}{5}$

$y = -\frac{6}{5}x + \frac{52}{5}$

5. Find the line perpendicular to the line containg the points (2, 7) and (-6, 5) that passes through the point (4, 1).

Slope of line segment = $\frac{5-7}{-6-2} = \frac{1}{4}$

Perpendicular slope = -4 point = (4, 1)

Perpendicular line: $y = mx + b$

$1 = (-4)(4) + b$ $\qquad$ $y = -4x + 17$

$1 = -16 + b$

$17 = b$

6. Solve the following inequalities. Show a number line analysis for each polynomial and rational inequality. Sketch a graph of each function on the graph paper provided and shade the solution to the inequality on the x-axis of the graph.

a) $3x + 7 \le 5x - 2$

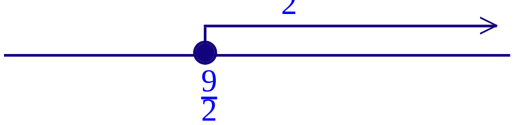
$3x + 7 \le 5x - 2$

$7 \le 2x - 2$

$9 \le 2x$

$\frac{9}{2} \le x$

$x \ge \frac{9}{2}$



Solution number line

Solution $\left[\frac{9}{2}, \infty\right)$

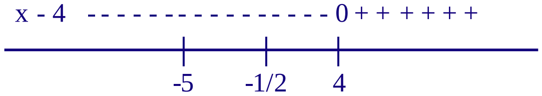
a) $(x - 4)(2x + 1)(x + 5) < 0$

$(x - 4)(2x + 1)(x + 5) < 0$ The product of three numbers is negative.

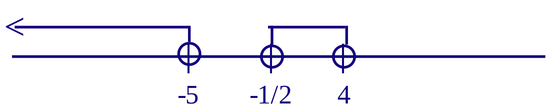
x + 5 - - - - - - 0 + + + + + + + + + + + + + +

2x + 1 - - - - - - - - - - - 0 + + + + + + + + + +

x - 4 - - - - - - - - - - - - - - - - 0 + + + + + + +



product - - - - - 0 + + 0 - - 0 + + + + + + +



Solution number line

Solution $(-\infty, -5) \vee \left(-\frac{1}{2}, 4\right)$

c) $\frac{x^2 - 9}{x + 2} \le 0$ Graph $y = \frac{x^2 - 9}{x + 2}$

$\frac{x^2 - 9}{x + 2} \le 0$ x + 3 - - - - - - 0 + + + + + + + + + + + + + + +

$\frac{(x - 3)(x + 3)}{x + 2} \le 0$ x + 2 - - - - - - - - - - - -0 + + + + + + + + + + + +

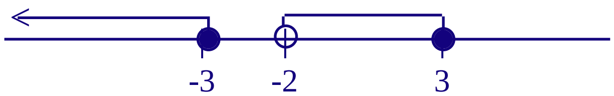
x - 3 - - - - - - - - - - - - - - - - - - - 0 + + + + + + +

quotient is neg or 0.

-3 -2 3

Quotient - - - - - - 0 + + ø - - - - - - - 0 + + + + + + + +

Solution # Line



Solution: $(-\infty, -3] \cup (-2, 3]$

d) $3x^2 - 10x - 8 \ge 0$ Graph $y = 3x^2 - 10x - 8$

$(3x + 2)(x - 4) \ge 0$ The product of two numbers is positive or zero.

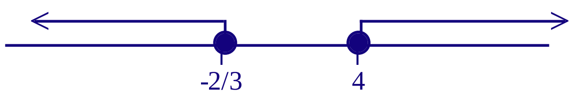
3x + 2 - - - - - - - 0 + + + + + + + + + + + + + +

x - 4 - - - - - - - - - - - - - - - - 0 + + + + + + + +

-2/3 4

product + + + + 0 - - - - - - 0 + + + + + +

Solution # Line



Solution: $\left(-\infty, -\frac{2}{3}\right] \cup [4, \infty)$

7. Solve each of the following equations. Leave all solutions in exact form and in approximate form. Check each one with your calculator.

a) $\frac{5}{x^2 + 5x + 4} = \frac{2x}{x^2 - 1}$

$$\frac{5}{(x + 4)(x + 1)} = \frac{2x}{(x + 1)(x - 1)}$$

$$\frac{5(x - 1)}{(x + 4)(x + 1)(x - 1)} = \frac{2x(x + 4)}{(x + 4)(x + 1)(x - 1)}$$

$$5(x - 1) = 2x(x + 4)$$

$$5x - 5 = 2x^2 + 8x$$

$$2x^2 + 3x + 5 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 40}}{4} = \frac{-3 \pm i\sqrt{31}}{4}$$

$$ss = \left\{ \frac{-3 \pm i\sqrt{31}}{4} \right\}$$

b) $x^3 + 3x^2 = 4x$

$$x^3 + 3x^2 - 4x = 0$$

$$x(x^2 + 3x - 4) = 0$$

$$x(x + 4)(x - 1) = 0$$

$x = 0$ or $x = -4$ or $x = 1$

SS = {-4, 0, 1}

c) $3x^2 - 4 = 2x$

$$3x^2 - 2x - 4 = 0$$

$$x = \frac{2 \pm \sqrt{4 - (-48)}}{6}$$

$$= \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm 2\sqrt{13}}{6}$$

$$= \frac{1 \pm \sqrt{13}}{3}$$

$$SS = \left\{\frac{1 \pm \sqrt{13}}{3}\right\}$$

d) $x^2 + 2x + 5 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

$$= \frac{-2 \pm \sqrt{-16}}{2}$$

$$= \frac{-2 \pm 4i}{2}$$

$$= -1 \pm 2i$$

$$SS = \{-1 \pm 2i\}$$

e) $\sqrt{2x+1} + x = 5$

$$\sqrt{2x+1} = 5 - x$$

$$(\sqrt{2x+1})^2 = (5 - x)^2$$

$$2x + 1 = x^2 - 10x + 25$$

$$x^2 - 12x + 24 = 0$$

$$x = \frac{12 \pm \sqrt{144 - 96}}{2} = \frac{12 \pm \sqrt{48}}{2}$$

$$x = \frac{12 \pm 4\sqrt{3}}{2} = 6 \pm 2\sqrt{3}$$

$$SS = \{ 6 \pm 2\sqrt{3} \}$$

8. Given the function $f(x) = 2x^2 + 5x$ and the function for g as shown in table form.
Determine the values of the following, simplify the algebra where needed:

x	**g(x)**
-4	13
-3	-20
-1	7
0	18
2	-5
4	8

i) $g(f(-1)) = g(-3)$
$= -20$

ii) $f(x + h) = 2(x + h)^2 + 5(x + h)$
$= 2(x^2 + 2xh + h^2) + 5x + 5h$
$= 2x^2 + 4xh + 2h^2 + 5x + 5h$

iii) $f(g(2)) = f(-5)$
$= 2(-5)^2 + 5(-5)$
$= 2(25) - 25$
$= 25$

iv) $f(x + h) - f(x) = (2x^2 + 4xh + 2h^2 + 5x + 5h) - (2x^2 + 5x)$
$= 2x^2 + 4xh + 2h^2 + 5x + 5h - 2x^2 - 5x$
$= 4xh + 2h^2 + 5h$

9. For each of the following functions, simplify $\frac{f(x+h)-f(x)}{h}$

a) $f(x) = 5x^2 + 4x - 7$

$$f(x + h) = 5(x + h)^2 + 4(x + h) - 7$$
$$= 5x^2 + 10xh + 5h^2 + 4x + 4h - 7$$
$$f(x + h) - f(x) = (5x^2 + 10xh + 5h^2 + 4x + 4h - 7) - (5x^2 + 4x - 7)$$
$$= 10xh + 5h^2 + 4h$$
$$\frac{f(x+h)-f(x)}{h} = \frac{10xh + 5h^2 + 4h}{h} = 10x + 5h + 4$$

b) $f(x) = \frac{2x}{x+1}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{2(x+h)}{x+h+1} - \frac{2x}{x+1}}{h} \cdot \frac{(x+h+1)(x+1)}{(x+h+1)(x+1)} = \frac{2(x+h)(x+1) - 2x(x+h+1)}{h(x+h+1)(x+1)} = \frac{2x^2 + 2x + 2xh + 2h - 2x^2 - 2xh - 2x}{h(x+h+1)(x+1)}$$
$$= \frac{2h}{h(x+h+1)(x+1)} = \frac{2}{(x+h+1)(x+1)}$$

10. Factor the following.

a) $3x^{-2} + 5x$

$$3x^{-2} + 5x = x^{-2}(3 + 5x^3)$$

b) $x^4 - 16$

$$x^4 - 16 = (x^2 - 4)(x^2 + 4)$$
$$= (x - 2)(x + 2)(x^2 + 4)$$

b) $x^2 - 9x - 22$

$$x^2 - 9x - 22 = (x - 11)(x + 2)$$

d) $6x^2 + 13x - 5$ ac = -30, b = 13, 15/-2

$$6x^2 + 13x - 5 = (3x - 1)(2x + 5)$$

e) $4x^3 - 12x^2 - 25x + 75$

$$4x^3 - 12x^2 - 25x + 75 = (4x^3 - 12x^2) + (-25x + 75)$$
$$= 4x^2(x - 3) - 25(x - 3)$$
$$= (x - 3)(4x^2 - 25)$$
$$= (x - 3)(2x - 5)(2x + 5)$$

f) $x^3 + 27$

$$x^3 + 27 = (x + 3)(x^2 - 3x + 9)$$

11. Jeff can paint a house in 30 hours working alone. Susan can paint the same house in 35 hours working alone. How long will it take to paint the house if they work together?

Jeff paints 1 house per 30 hours $\rightarrow \frac{1}{30}$ house per hour

Susan paints 1 house per 35 hours $\rightarrow \frac{1}{35}$ house per hour

Together they paint 1 house per x hours $\rightarrow \frac{1}{x}$ house per hour

$$\frac{1}{30} + \frac{1}{35} = \frac{1}{x}$$
$$\frac{13}{210} = \frac{1}{x}$$
$$x = \frac{210}{13} \approx 16.15$$

Together they can paint the house in about 16 hours 9 minutes.

12. Brett wants to make 1 ton of 15% protein ration for his sheep by mixing together soybean meal and whole corn. If soybean meal is 44% protein and whole corn is 8.5% protein, how many pounds of each will he use? (Hint: 1 ton = 2000 pounds)

	% protein	# lbs mix	# lbs protein
meal	.44	x	.44x
corn	.085	y	.085y
mixture	.15	2000	300

$$x + y = 2000 \rightarrow -85x - 85y = -170000$$
$$.44x + .085y = 300 \rightarrow 440x + 85y = 300000$$
$$355x = 130000$$
$$x \approx 366.2$$

Brett should mix together 366.2 pounds of soybean meal and 1633.8 pounds of corn.

13. Rabbits in a lab are to be kept on a strict daily diet to include 30 grams of protein, 16 grams of fat, and 24 grams of carbohydrates. The scientist has only three food mixes available with the following grams of nutrients per unit.

	Protein	**Fat**	**Carbohydrates**
Mix A	4	6	3
Mix B	6	1	2
Mix C	4	1	12

Find how many units of each mix are needed daily to meet one rabbit's dietary needs.

Let A = # units of mix A and let B = # units of mix B and let C = # units of mix C.

protein: $4A + 6B + 4C = 30$

fat: $6A + B + C = 16$

carbs: $3A + 2B + 12C = 24$

$$\begin{bmatrix} 4 & 6 & 4 & 30 \\ 6 & 1 & 1 & 16 \\ 3 & 2 & 12 & 24 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$A = 2, B = 3, C = 1$

Scientists should feed each rabbit 2 units of mix A, 3 units of mix B, and 1 unit of mix C.

14. Simplify:

a) $\sqrt{192x^3y^6}$

$$\sqrt{192x^3y^6} = \sqrt{2^6 \cdot 3x^3y^6}$$
$$= 2^3xy^3\sqrt{3x}$$
$$= 8xy^3\sqrt{3x}$$

b) $\sqrt[5]{160x^7y^8}$

$$\sqrt[5]{160x^7y^8} = \sqrt[5]{2^5 \cdot 5x^7y^8}$$
$$= 2xy\sqrt[5]{5x^2y^3}$$

15. Simplify the following by hand. Explain the process you used.

a) $27^{2/3}$

$27^{2/3} = 3^2 = 9$

First cube root the 27 to get 3 then square the 3 to get 9.

b) $16^{3/4}$

$16^{3/4} = 2^3 = 8$

First 4th root the 16 to get 2 then cube the 2 to get 8.

16. Simplify: $2\sqrt{27} + \sqrt{12} + 3\sqrt{75}$

$$\begin{aligned} 2\sqrt{27} + \sqrt{12} + 3\sqrt{75} &= 2\sqrt{3^3} + \sqrt{2^2 \cdot 3} + 3\sqrt{3 \cdot 5^2} \\ &= 2 \cdot 3\sqrt{3} + 2\sqrt{3} + 3 \cdot 5\sqrt{3} \\ &= 6\sqrt{3} + 2\sqrt{3} + 15\sqrt{3} \\ &= 23\sqrt{3} \end{aligned}$$

17. If pipe A can fill a pool half as much time as it takes pipe B to fill the same pool. If it takes 8 hours to fill the pool using both pipes, how long does it take with each pipe alone?

Pipe A fills 1 pool per x hours $\rightarrow \frac{1}{x}$ pools per hour

Pipe B fills 1 pool per 2x hours $\rightarrow \frac{1}{2x}$ pools per hour

Together they fill 1 pool per 8 hours $\rightarrow \frac{1}{8}$ pools per hour

$$\begin{aligned} \frac{1}{x} + \frac{1}{2x} &= \frac{1}{8} \\ \frac{8}{8x} + \frac{4}{8x} &= \frac{x}{8x} \\ 8 + 4 &= x \\ x &= 12 \end{aligned}$$

Pipe A fills the pool in 12 hours and pipe b fills the pool in 24 hours.

18. Add, subtract, multiply, or divide as indicated.

a) $\frac{3x+2}{x^2+3x-10} + \frac{5x}{x^2-25}$

$$\begin{aligned} \frac{3x+2}{x^2+3x-10} + \frac{5x}{x^2-25} &= \frac{3x+2}{(x+5)(x-2)} + \frac{5x}{(x+5)(x-5)} \\ &= \frac{(3x+2)(x-5)}{(x+5)(x-2)(x-5)} + \frac{5x(x-2)}{(x+5)(x-2)(x-5)} \\ &= \frac{(3x+2)(x-5) + 5x(x-2)}{(x+5)(x-2)(x-5)} \\ &= \frac{3x^2 - 15x + 2x - 10 + 5x^2 - 10x}{(x+5)(x-2)(x-5)} \\ &= \frac{8x^2 - 23x - 10}{(x+5)(x-2)(x-5)} \end{aligned}$$

b) $\dfrac{x-3}{x^2+5x-6} - \dfrac{x+1}{x^2-2x+1}$

$$\frac{x-3}{x^2+5x-6} - \frac{x+1}{x^2-2x+1} = \frac{x-3}{(x+6)(x-1)} - \frac{x+1}{(x-1)(x-1)}$$

$$= \frac{(x-3)(x-1)}{(x+6)(x-1)(x-1)} - \frac{(x+1)(x+6)}{(x+6)(x-1)(x-1)}$$

$$= \frac{(x-3)(x-1)-(x+1)(x+6)}{(x+6)(x-1)(x-1)}$$

$$= \frac{(x^2-4x+3)-(x^2+7x+6)}{(x+6)(x-1)(x-1)}$$

$$= \frac{x^2-4x+3-x^2-7x-6}{(x+6)(x-1)(x-1)}$$

$$= \frac{-11x-3}{(x+6)(x-1)(x-1)}$$

c) $\dfrac{x^2-3x-10}{x^2+5x+6} \cdot \dfrac{x^2+6x+9}{x^2-2x-15}$

$$\frac{x^2-3x-10}{x^2+5x+6} \cdot \frac{x^2+6x+9}{x^2-2x-15} = \frac{(x-5)(x+2)}{(x+2)(x+3)} \cdot \frac{(x+3)(x+3)}{(x+3)(x-5)}$$

$$= \frac{(x-5)}{(x+3)} \cdot \frac{(x+3)}{(x-5)}$$

$$= \frac{(x-5)(x+3)}{(x+3)(x-5)}$$

$$= 1$$

d) $\dfrac{x^2+x}{x^2-1} \div \dfrac{3x}{x^2+4}$

$$\frac{x^2+x}{x^2-1} \div \frac{3x}{x^2+4} = \frac{x(x+1)}{(x+1)(x-1)} \div \frac{3x}{\left(x^2+4\right)}$$

$$= \frac{x}{(x-1)} \cdot \frac{\left(x^2+4\right)}{3x}$$

$$= \frac{x\left(x^2+4\right)}{3x(x-1)}$$

$$= \frac{\left(x^2+4\right)}{3(x-1)}$$